Weighted DOP With Consideration on Elevation-Dependent Range Errors of GNSS Satellites

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***Abstract*—This paper proposes the weighted dilution of precision (WDOP) with consideration of the satellite elevation angle in order to improve the performance of dilution of precision (DOP), which is a standard tool to quantify the positional precision of the Global Navigation Satellite System (GNSS). The WDOP is calculated by assigning different weights to visible GNSS satellites depending on their elevation angles. In order to demonstrate the effectiveness of WDOP, the conventional DOP and WDOP were mathematically analyzed and a comparative analysis was conducted using actual Global Positioning System data. Results showed that WDOP represents the position error trends more accurate than the conventional DOP, particularly when low-elevation measurements were used for positioning calculation. Therefore, the WDOP could be a promising replacement of DOP as a tool for representing and quantifying errors in GNSS positioning.**

***Index Terms*—Covariance matrix, estimation, Global Positioning System (GPS), least squares methods, satellite navigation system.**

I. Introduction

THE Global Navigation Satellite System (GNSS) provides precise positioning worldwide and has various application areas such as the navigation of automobiles [1], ships [2], and airplanes [3], along with time synchronization [4], cartography, and geodesy [1], [5].

In principles, GNSS converts the time difference between signal transmission of the satellite and signal reception of the receiver into an equivalent range [5]–[7], which can be utilized to compute the user position based on trilateration. The accuracy of GNSS positioning depends on the visible satellite constellation and measurement errors. GNSS community uses a concept called dilution of precision (DOP) [5]–[7] to represent the geometric formation of visible satellites [8], [9]. Generally, the DOP value is large when the satellite formation is densely distributed and is small when the formation is widely distributed. In addition, the DOP tends to be inversely proportional to the user position error and is used in combination with the user-equivalent range error (UERE) as a standard tool to infer and predict the user position error [10], [11]. On the basis of these characteristics, DOP has become the evaluation standard for GNSS position error analysis since the initial development stages of GNSS [5], [6].

However, the DOP does not always reflect the GNSS position error faithfully [12] as it is a parameter that considers only the geometric formation of satellites. When the satellite formation becomes more widely and evenly distributed as lower elevation satellites are included, the DOP is decreased and the position error indicated by the DOP value is reduced also owing to its proportional relation to DOP [12], [13]. However, for low-elevation satellites, the time duration of the GNSS signal passing through the ionosphere and the troposphere is increased [5], [14], leading to an increment of the position errors, which is contradictory to the DOP changes [6], [7], [12]. Hence, the relationship between DOP and position error cannot be simply described for the satellite formation of equivalent DOP [12], [15] as the position error increases as more low-elevation angle satellites are included. Therefore, the conventional DOP has limitations in describing the position error since DOP takes into consideration only the satellite geometric formation and neglects signal transmission-related errors, which become dominant for low-elevation satellites. The limitations of DOP are discussed in detail in Section III of this paper.

Various studies have been pursued to overcome the inability of DOP to faithfully reflect the effect of satellite elevation angle on the position error. Sairo *et al.* [15] expanded upon the weighted least squares method to calculate the covariance and divided by σ_{UERE} (sigma of UERE) in order to calculate the weighted DOP (WDOP). In this case, the WDOP was analyzed with respect to the number of visible satellites, but no calculations reflecting the satellite elevation angles were performed. Additionally, an analysis focused on satellite

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selection/exclusion for Assisted Global Positioning System management was conducted without considering how to overcome the limitations of the conventional DOP. Li and Wu [16] applied various error models to set the weight value in the weighted least squares method in order to obtain accurate position values and analyzed the performance. Although the WDOP was defined by taking partial equations among the calculations, carrier-to-noise ratio (C/N0) was used rather than the satellite elevation angle as the accurate position value calculation was the main objective. Leva [17] used the weighted least squares method to compute vertical position error and vertical DOP (VDOP); however, this work was focused on the correlation analysis between the vertical error and VDOP and did not address the effects of satellite elevation angles. Such previous research has been more concerned with the improvement of the position accuracy while the properties and characteristics of the WDOP and the limitations of the DOP were not sufficiently investigated [15]–[17]. For instance, in [16], the weight was calculated by using the more realistic receiver measurement value C/N0 rather than satellite elevation angles. C/N0 is a parameter that changes due to the surrounding environmental factors. Although C/N0 can be used for improving the position value, it irregularly changes by the surrounding environment. Furthermore, it is not readily applicable as an assessment standard that presents overall performance trends such as variation of the measurement error depending on the satellite elevation angle.

This paper proposes the WDOP that considers the satellite elevation angles in order to overcome the limitations of the conventional DOP that are due to the lack of consideration for the error changes according to satellite elevation angles. In the WDOP calculation, the measurement error model that reflects the satellite elevation angle was normalized using σ_{UERE} to be used as the weight value. WDOP was mathematically derived, and its characteristics were analyzed. Furthermore, the ability of the proposed WDOP to reflect the position error trends was verified by analyzing actual Global Positioning System (GPS) data. Verification was conducted by calculating the conventional DOP and WDOP followed by a comparative analysis of the position error trends.

In Section II of this paper, the generally used GPS observation model is shown. The conventional DOP calculated from the GPS observation model is analyzed in Section III. In Section IV, the proposed WDOP is derived, and its characteristics are analyzed. The analysis results with collected data using the proposed WDOP are shown in Section V, and the paper is concluded in Section VI.

II. GPS Observation Model

The pseudorange measurement value of GNSS is calculated from the propagation time difference of the transmitted navigation signal from the satellite to the receiver (user). The distance between the satellite and the receiver can be described by the geometric relationship between the satellite position and the user position along with clock bias, which is the time difference between the satellite and the receiver clocks [5]. Generally, the satellite clock bias can be corrected using the transmitted navigation message; hence, assuming that only the receiver clock bias term exists, the following observation equation of (1) is obtained [5]–[7]:

$$\rho_u^s = \sqrt{(x^s - x_u)^2 + (y^s - y_u)^2 + (z^s - z_u)^2} + c\Delta b_u \quad (1)$$

where

ρ_u^s *measured pseudorange*;
$\mathbf{x}^s$ $= [x^s \quad y^s \quad z^s]^T$. *Satellite position*;
$\mathbf{x}_u$ $= [x_u \quad y_u \quad z_u]^T$. *User position*;
c *speed of light*;
Δb_u *clock bias of the receiver*.

Linearizing (1) with respect to the nominal user position gives (2). Expanding this for all visible satellites and rearranging the equation gives (3) [5] as follows:

$$\rho_u^s = \rho_{u0}^s - \frac{x^s - x_{u0}}{\rho_{u0}^s}\Delta x_u - \frac{y^s - y_{u0}}{\rho_{u0}^s}\Delta y_u - \frac{z^s - z_{u0}}{\rho_{u0}^s}\Delta z_u + c\Delta b_u \quad (2)$$

$$\underbrace{\begin{bmatrix} \rho_u^1 - \rho_{u0}^1 \\ \rho_u^2 - \rho_{u0}^2 \\ \vdots \\ \rho_u^N - \rho_{u0}^N \end{bmatrix}}_{\Delta y} = \underbrace{\begin{bmatrix} -\frac{x^1 - x_{u0}}{\rho_{u0}^1} & -\frac{y^1 - y_{u0}}{\rho_{u0}^1} & -\frac{z^1 - z_{u0}}{\rho_{u0}^1} & 1 \\ -\frac{x^2 - x_{u0}}{\rho_{u0}^2} & -\frac{y^2 - y_{u0}}{\rho_{u0}^2} & -\frac{z^2 - z_{u0}}{\rho_{u0}^2} & 1 \\ & \vdots & & \\ -\frac{x^N - x_{u0}}{\rho_{u0}^N} & -\frac{y^N - y_{u0}}{\rho_{u0}^N} & -\frac{z^N - z_{u0}}{\rho_{u0}^N} & 1 \end{bmatrix}}_{H} \times \underbrace{\begin{bmatrix} \Delta x_u \\ \Delta y_u \\ \Delta z_u \\ c\Delta b_u \end{bmatrix}}_{\Delta \mathbf{x}_u} \quad (3)$$

$$\Delta y = H\Delta\mathbf{x}_u \quad (4)$$

where

$\mathbf{x}_{u0}$ $= [\, x_{u0} \quad y_{u0} \quad z_{u0} \quad c\Delta b_{u0} \,]^T$;
ρ_{u0}^s $= \sqrt{(x^s - x_{u0})^2 + (y^s - y_{u0})^2 + (z^s - z_{u0})^2} + c\Delta b_{u0}$;
N *total number of visible satellites.*

Estimating the position corrections using the simple estimation method of least squares, the user position can be estimated by compensating the nominal point

$$\Delta\hat{\mathbf{x}}_u = (H^T H)^{-1} H^T \Delta y \quad (5)$$

$$\hat{\mathbf{x}}_u = \mathbf{x}_{u0} + \Delta\hat{\mathbf{x}}_u. \quad (6)$$

III. DOP Characteristics

DOP is a multiplicative parameter that converts the pseudorange measurement error to the user position coordinates for a given satellite formation. DOP is derived from the error covariance, which is calculated from the least squares estimation equation used in Section II [5], [18]. When the user position covariance is rearranged as in (8), (9) is obtained from a simple multiplication of σ_{UERE}^2 and combination of the observation matrix. For convenience, subscript "u," which refers to the user position, is omitted

$$\Delta\hat{\mathbf{x}} = (H^T H)^{-1} H^T \Delta y \tag{7}$$

$$\begin{aligned} E[\Delta\hat{\mathbf{x}}\Delta\hat{\mathbf{x}}^T] &= E\left[(H^T H)^{-1} H^T \Delta y \Delta y^T H (H^T H)^{-1}\right] \\ &= (H^T H)^{-1} H^T \underbrace{E[\Delta y \Delta y^T]}_{=R=\sigma^2_{\mathrm{UERE}} I} H (H^T H)^{-1} \\ &= (H^T H)^{-1} H^T \left[\sigma^2_{\mathrm{UERE}} I\right] H (H^T H)^{-1} \\ &= (H^T H)^{-1} H^T H (H^T H)^{-1} \cdot \sigma^2_{\mathrm{UERE}} \\ &= (H^T H)^{-1} \cdot \sigma^2_{\mathrm{UERE}} \end{aligned} \tag{8}$$

$$\mathrm{Cov}[\Delta\hat{\mathbf{x}}] = (H^T H)^{-1} \cdot \sigma^2_{\mathrm{UERE}}. \tag{9}$$

Equation (9) shows the projection of the pseudorange measurement error onto the position error. Here, each component of the calculated covariance can be arranged as given in (10), and the covariance of each axial direction can be obtained by extracting the diagonal elements

$$\mathrm{Cov}[\Delta\hat{\mathbf{x}}] = \begin{bmatrix} E[\Delta x^2] & E[\Delta x \Delta y] & E[\Delta x \Delta z] & E[\Delta x c\cdot\Delta b] \\ E[\Delta y \Delta x] & E[\Delta y^2] & E[\Delta y \Delta z] & E[\Delta y c\cdot\Delta b] \\ E[\Delta z \Delta x] & E[\Delta z \Delta y] & E[\Delta z^2] & E[\Delta z c\cdot\Delta b] \\ E[c\cdot\Delta b \Delta x] & E[c\cdot\Delta b \Delta y] & E[c\cdot\Delta b \Delta z] & E[c^2\cdot\Delta b^2] \end{bmatrix}. \tag{10}$$

The covariance of each axial direction can be described with (12), which is the product of the diagonal elements of matrix D defined in (11) and σ^2_{UERE}

$$D = (H^T H)^{-1} \tag{11}$$

$$\left.\begin{aligned} E[\Delta x^2] &= D_{11}\cdot\sigma^2_{\mathrm{UERE}} \\ E[\Delta y^2] &= D_{22}\cdot\sigma^2_{\mathrm{UERE}} \\ E[\Delta z^2] &= D_{33}\cdot\sigma^2_{\mathrm{UERE}} \\ E[c^2\cdot\Delta b^2] &= D_{44}\cdot\sigma^2_{\mathrm{UERE}} \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} \sigma_x &= \sqrt{D_{11}}\cdot\sigma_{\mathrm{UERE}} \\ \sigma_y &= \sqrt{D_{22}}\cdot\sigma_{\mathrm{UERE}} \\ \sigma_z &= \sqrt{D_{33}}\cdot\sigma_{\mathrm{UERE}} \\ c\cdot\sigma_b &= \sqrt{D_{44}}\cdot\sigma_{\mathrm{UERE}}. \end{aligned}\right. \tag{12}$$

According to the general definition of DOP, (13) and (14) can be derived. The DOP, which represents the satellite formation, is widely utilized in satellite constellation design and navigation performance prediction [5], [6], [18]

$$\mathrm{DOP} = \begin{cases} \text{Geometric DOP(GDOP)} \\ \quad = \sqrt{D_{11} + D_{22} + D_{33} + D_{44}} \\ \text{Position DOP(PDOP)} \\ \quad = \sqrt{D_{11} + D_{22} + D_{33}} \\ \text{Horizontal DOP(HDOP)} \\ \quad = \sqrt{D_{11} + D_{22}} \\ \text{Vertical DOP(VDOP)} = \sqrt{D_{33}} \\ \text{Time DOP(TDOP)} = \sqrt{D_{44}} \end{cases} \tag{13}$$

$$\sigma_{\mathbf{x}} = \mathrm{DOP} \cdot \sigma_{\mathrm{UERE}}. \tag{14}$$

Here, D_{ii} refers to the ith row and the column diagonal element of matrix D. The UERE in GNSS generally increases with lower satellite elevation angles, and thus, the position estimation error increases as lower elevation angle satellites are included [5], [6], [14]. However, as more low-elevation satellites are included, the relative geometry between the satellites and the receiver is improved, and thus, the DOP decreases. The reduction of DOP signifies the reduction of the position estimation error due to (14), and in real situations, there are cases where the opposite occurs. As the DOP does not satisfy the position error changes affected by the satellite elevation angles, a system that employs a standard based on (14) has the limitation of not being able to present the error trends accurately. Therefore, this paper proposes the WDOP that calculates the DOP by setting the weight to reflect the position error trends for low elevation angles. Fig. 1 conceptually illustrates the discussed limitation and proposed the DOP, showing the DOP, WDOP, and position error change trends as the elevation angle of four visible satellites is gradually reduced. The lower elevation angle becomes the larger pseudorange error that occurs; hence, the position error is not continuously decreased. As shown in the figure, the WDOP that reflects the position error trends in the region, including visible satellites with low elevation angles, is proposed. The detailed derivation and an explanation of the proposed WDOP are dealt with in Section IV.

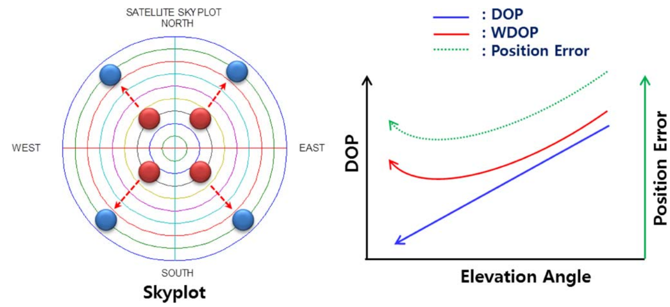

Fig. 1. (Right) DOP, WDOP, and position error changes according to (left) satellite elevation angle changes.

IV. WDOP Derivation and Analysis

A. WDOP Derivation

WDOP begins with the weighted least squares method where the position value is calculated by assigning different weight values according to the included error magnitude for each satellite. The general weighted least squares method sets the weight value as the inverse of the error covariance matrix of pseudorange measurements that is denoted as R in (16) and (17). R includes both effects of range errors with respect to elevation angle and inherent random noise. In this paper, R was estimated based on the collected pseudorange measurements and normalized with σ^2_{UERE}, as shown in (16), to simplify the results. It should be noted that σ^2_{UERE} is generally regarded as a constant and is a representative value for all satellites [19]. Equation (17) shows a relationship between the weight and R

$$\Delta\hat{\mathbf{x}} = (H^T W H)^{-1} H^T W \Delta y \tag{15}$$

$$W = \begin{bmatrix} w_1 & 0 & & 0 \\ 0 & w_2 & \cdots & 0 \\ & \vdots & \ddots & \\ 0 & 0 & & w_N \end{bmatrix}, \quad w_i = \left(\frac{R_i}{\sigma^2_{\mathrm{UERE}}}\right)^{-1}$$

N : satellite index

R_i : i^{th} pseudorange covariance (16)

$$W = \left(\frac{1}{\sigma^2_{\mathrm{UERE}}} R\right)^{-1}. \tag{17}$$

Applying the weighted least squares equation to (8) and rearranging gives the equation to calculate the position error covariance. The bottommost equation in (18), shown at the bottom of the page, is described with matrix D, which can calculate the DOP. The diagonal components of matrix D in (19), shown at the bottom of the page, can compute the WDOP, as illustrated in (13).

B. Weight Model

An appropriate weight value has to be set in order to have the previously derived WDOP that reflects the effect of satellite elevation angle θ. The pseudorange error model according to satellite elevation angle has been defined in various studies [16], [20]. In reference to established research results, the pseudorange error model can be described with the addition of constant terms and terms additionally occurring from the elevation angle changes. Generally, the error is smallest when the satellite is located at the zenith (elevation angle of 90°) [5], [6]. If this value is σ^2_{UERE}, the pseudorange error can be represented with (20). $f(\theta)$ is introduced to express the pseudorange errors σ_θ that are not included in σ^2_{UERE}. It accounts for additional range errors when the elevation angle is decreased from 90°. In this paper, σ_θ instead of $f(\theta)$ is estimated from a set of pseudorange measurements and will be given in Section V. Note that the estimated σ^2_θ will be used to compute R matrix

$$\sigma^2_\theta = \sigma^2_{90^\circ} + f(\theta) \tag{20}$$

$$\sigma^2_\theta = \sigma^2_{\mathrm{UERE}} + f(\theta). \tag{21}$$

C. WDOP Characteristics Analysis

Theorem 1 represents the magnitude analysis results of the conventional DOP and WDOP using (21).

Theorem 1: The WDOP is always greater than the DOP if the pseudorange error of the GNSS satisfies the following inequality:

$$\text{If } \sigma^2_\theta = \sigma^2_{\mathrm{UERE}} + f(\theta) \text{ and } f(\theta) \geq 0, \text{ then WDOP} \geq \text{DOP}$$

where

$$\sigma^2_{\mathrm{UERE}} = \sigma^2_{90^\circ} \qquad \theta : \text{Elevation angle.}$$

Proof: According to the definition of WDOP, the weight can be described as $w_i = [R_i/\sigma^2_{\mathrm{UERE}}]^{-1} = [\sigma^2_{\mathrm{UERE}}/\sigma^2_{\mathrm{UERE}} + f(\theta_i)]$. Here, since $\sigma^2_{\mathrm{UERE}} > 0$, $f(\theta) \geq 0$, the range of the weight can be set to $w_i \leq 1$. Using inequality properties [21], [22], comparing the magnitude to calculate the WDOP and DOP

$$\|H^T W H\| \leq \|H^T H\|$$

where "$\|\cdot\|$" takes the norm of its content.

Taking the reciprocal of both sides and changing the above into the DOP calculation form

$$\left\|(H^T W H)^{-1}\right\| \geq \left\|(H^T H)^{-1}\right\|$$

$$\therefore \ \text{WDOP} \geq \text{DOP}.$$

■

In the case when the additional term among the measurement value covariance for all visible satellites in Theorem 1 is 0 or $f(\theta) = 0$, the WDOP and the DOP are equal. However, in order to obtain a value of 0, all the satellites must have an elevation angle of 90°, which is practically impossible. Therefore, the defined WDOP is always greater than the DOP. In observation of this mathematical characteristic, the conventional DOP tends to be optimistic because it relatively poorly reflects the effect of satellite elevation angle.

To simplify the problem, the case of four observed satellites is used first. After calculating the WDOP defined in (19) for the four pseudorange measurement values, the East, North, and Up directional components of WDOP (diagonal elements) can be represented through (23). Looking at (23), the $\vec{D}_{\mathrm{WDOP}}$ component is described as the product $(\tilde{W})$ of matrix A composed of the line-of-sight vectors and the weight reciprocal. When

$$\begin{aligned} E[\Delta\hat{x}\,\Delta\hat{x}^T] &= E\left[(H^T W H)^{-1} H^T W \underbrace{\Delta y \Delta y^T}_{E[\Delta y \Delta y^T] = \sigma^2_{\mathrm{UERE}} W^{-1}} W^T H (H^T W H)^{-1}\right] \\ &= (H^T W H)^{-1} H^T W \sigma^2_{\mathrm{UERE}} W^{-1} W^T H (H^T W H)^{-1} \\ &= \sigma^2_{\mathrm{UERE}} (H^T W H)^{-1} H^T W H (H^T W H)^{-1} \\ &= \sigma^2_{\mathrm{UERE}} (H^T W H)^{-1} \end{aligned} \tag{18}$$

$$D_{\mathrm{WDOP}} = (H^T W H)^{-1} \tag{19}$$

$\tilde{W} = \vec{1}$, the equation is equivalent to the equation to calculate each component of the DOP ($\vec{D}_{\mathrm{DOP}}$)

$$H = \begin{bmatrix} e_1 & n_1 & u_1 & 1 \\ e_2 & n_2 & u_2 & 1 \\ e_3 & n_3 & u_3 & 1 \\ e_4 & n_4 & u_4 & 1 \end{bmatrix} \qquad W = \begin{bmatrix} w_1 & 0 & 0 & 0 \\ 0 & w_2 & 0 & 0 \\ 0 & 0 & w_3 & 0 \\ 0 & 0 & 0 & w_4 \end{bmatrix} \tag{22}$$

$$\begin{aligned} D_{11} =& \frac{1}{Den} \\ &\times\Bigg(\frac{1}{w_1}(n_3u_2 - n_4u_2 - n_2u_3 + n_4u_3 + n_2u_4 - n_3u_4)^2 \\ &+ \frac{1}{w_2}(n_3u_1 - n_4u_1 - n_1u_3 + n_4u_3 + n_1u_4 - n_3u_4)^2 \\ &+ \frac{1}{w_3}(n_2u_1 - n_4u_1 - n_1u_2 + n_4u_2 + n_1u_4 - n_2u_4)^2 \\ &+ \frac{1}{w_4}(n_2u_1 - n_3u_1 - n_1u_2 + n_3u_2 + n_1u_3 - n_2u_3)^2\Bigg) \\ D_{22} =& \frac{1}{Den} \\ &\times\Bigg(\frac{1}{w_1}(e_3u_2 - e_4u_2 - e_2u_3 + e_4u_3 + e_2u_4 - e_3u_4)^2 \\ &+ \frac{1}{w_2}(e_3u_1 - e_4u_1 - e_1u_3 + e_4u_3 + e_1u_4 - e_3u_4)^2 \\ &+ \frac{1}{w_3}(e_2u_1 - e_4u_1 - e_1u_2 + e_4u_2 + e_1u_4 - e_2u_4)^2 \\ &+ \frac{1}{w_4}(e_2u_1 - e_3u_1 - e_1u_2 + e_3u_2 + e_1u_3 - e_2u_3)^2\Bigg) \\ D_{33} =& \frac{1}{Den} \\ &\times\Bigg(\frac{1}{w_1}(e_3n_2 - e_4n_2 - e_2n_3 + e_4n_3 + e_2n_4 - e_3n_4)^2 \\ &+ \frac{1}{w_2}(e_3n_1 - e_4n_1 - e_1n_3 + e_4n_3 + e_1n_4 - e_3n_4)^2 \\ &+ \frac{1}{w_3}(e_2n_1 - e_4n_1 - e_1n_2 + e_4n_2 + e_1n_4 - e_2n_4)^2 \\ &+ \frac{1}{w_4}(e_2n_1 - e_3n_1 - e_1n_2 + e_3n_2 + e_1n_3 - e_2n_3)^2\Bigg) \\ Den =& \frac{\text{Determinant of } H^TWH}{w_1w_2w_3w_4}. \end{aligned} \tag{23}$$

Equation (24) is obtained by using component a_{ij} combining the weight and the line-of-sight vector to rearrange each component in (23)

$$\vec{D}_{\mathrm{WDOP}} = \begin{bmatrix} D_{11} \\ D_{22} \\ D_{33} \end{bmatrix}_{\mathrm{WDOP}} = \underbrace{\begin{bmatrix} a_{e_1} & a_{e_2} & a_{e_3} & a_{e_4} \\ a_{n_1} & a_{n_2} & a_{n_3} & a_{n_4} \\ a_{u_1} & a_{u_2} & a_{u_3} & a_{u_4} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \frac{1}{w_1} \\ \frac{1}{w_2} \\ \frac{1}{w_3} \\ \frac{1}{w_4} \end{bmatrix}}_{\tilde{W}} \tag{24}$$

$$\vec{D}_{\mathrm{DOP}} = \begin{bmatrix} D_{11} \\ D_{22} \\ D_{33} \end{bmatrix}_{\mathrm{DOP}} = \begin{bmatrix} a_{e_1} + a_{e_2} + a_{e_3} + a_{e_4} \\ a_{n_1} + a_{n_2} + a_{n_3} + a_{n_4} \\ a_{u_1} + a_{u_2} + a_{u_3} + a_{u_4} \end{bmatrix} = A\vec{1}_{4\times 1}. \tag{25}$$

Theorem 2: The WDOP can be described as the DOP product of correction coefficient

$$\vec{D}_{\mathrm{WDOP}} = (1 + C)\vec{D}_{\mathrm{DOP}}, \quad 0 \le C.$$

Proof: The result of (25) can be expanded to the case of where more than four satellites are observed

$$H = \begin{bmatrix} e_1 & n_1 & u_1 & 1 \\ e_2 & n_2 & u_2 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ e_N & n_N & u_N & 1 \end{bmatrix}$$

$$W = \begin{bmatrix} w_1 & 0 & 0 & 0 \\ 0 & w_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & w_N \end{bmatrix}$$

$$A = \begin{bmatrix} a_e \\ a_n \\ a_u \end{bmatrix} = \begin{bmatrix} a_{e_1} & a_{e_2} & \cdots & a_{e_N} \\ a_{n_1} & a_{n_2} & \cdots & a_{n_N} \\ a_{u_1} & a_{u_2} & \cdots & a_{u_N} \end{bmatrix}$$

$$\tilde{W} = \begin{bmatrix} \frac{1}{w_1} & \frac{1}{w_2} & \cdots & \frac{1}{w_N} \end{bmatrix}^T$$

$$\vec{D}_{\mathrm{WDOP}} = A\tilde{W}$$

$$\text{if} \quad \tilde{W} = \vec{1}, \quad \text{then} \quad \vec{D}_{\mathrm{WDOP}} = \vec{D}_{\mathrm{DOP}} (= A\vec{1}).$$

Rearranging $\tilde{W}$ using Theorem 1

$$\tilde{w}_i = \frac{1}{w_i} = \frac{\sigma^2_{\mathrm{UERE}} + f(\theta_i)}{\sigma^2_{\mathrm{UERE}}} = 1 + \frac{f(\theta_i)}{\sigma^2_{\mathrm{UERE}}} = 1 + \Sigma_i,$$

$$\Sigma_i = \frac{f(\theta_i)}{\sigma^2_{\mathrm{UERE}}}$$

$$\begin{aligned} \tilde{W}_{N\times 1} &= [1 + \Sigma_1 \quad 1 + \Sigma_2 \quad \cdots \quad 1 + \Sigma_N]^T \\ &= \vec{1} + \vec{\Sigma}. \end{aligned}$$

From this, $\vec{D}_{\mathrm{WDOP}}$ becomes $\vec{D}_{\mathrm{WDOP}} = A(\vec{1} + \vec{\Sigma})$, and since $A\vec{1} = \vec{D}_{\mathrm{DOP}}$, it can be restated as $\vec{D}_{\mathrm{WDOP}} = \vec{D}_{\mathrm{DOP}} + A\vec{\Sigma}$. Therefore, $\vec{D}_{\mathrm{WDOP}} = (1 + C)\vec{D}_{\mathrm{DOP}}$ is established

$$\vec{D}_{\mathrm{WDOP}} = A(\vec{1} + \vec{\Sigma}) = \underbrace{A\vec{1}}_{=\vec{D}_{\mathrm{DOP}}} + A\vec{\Sigma} = \vec{D}_{\mathrm{DOP}} + A\vec{\Sigma}.$$

■

Fig. 2. Location of GPS antenna in Jeju International Airport.

V. Experiment and Analysis

A. Experiment Environment

To verify that the proposed WDOP reflects the position error trends, a set of GPS data was analyzed. The GPS data were collected with a Novatel DLV-3 (OEM-V card) receiver at the Jeju International Airport for 2 h on June 10, 2009. The receiver antenna was located on the rooftop of an office building where the exact position was known *a priori* (see Fig. 2). MATLAB R2010a was used for the data analysis. As mentioned in Section IV, the pseudorange error model was estimated from the collected GPS data.

B. Weight Setting

In this paper, the assumed pseudorange measurement error was configured to increase with a decrease in satellite elevation angle. After observing this phenomenon, the pseudorange error depending on the visible satellite elevation angle was analyzed in the same environment for 24 h in order to obtain the weight value to be used in WDOP. As the antenna used for data collection was installed at the known location, the pseudorange error between the satellite and the receiver could be calculated. The satellite clock bias was computed from the satellite navigation message and removed during the error calculation.

Fig. 3 shows the pseudorange error depending on the satellite elevation angle. The overall trend reveals that the error is slightly reduced when the elevation angle is above 30°, whereas it rapidly increased when the elevation angle was below 30°. Therefore, the pseudorange error increases as more low-elevation satellites are included, reducing the positioning accuracy. This trend is not reflected in the conventional DOP, and thus, this paper seeks to propose the WDOP that reflects the position error trends. The pseudorange error was modeled using the exponential model or function based on the analysis results and used in the WDOP calculation. The pseudorange error model can be modeled in various forms other than the exponential model and can change depending on the user environmental factors.

The pseudorange error model was estimated by MATLAB Curve Fitting Toolbox in which a first-order exponential model

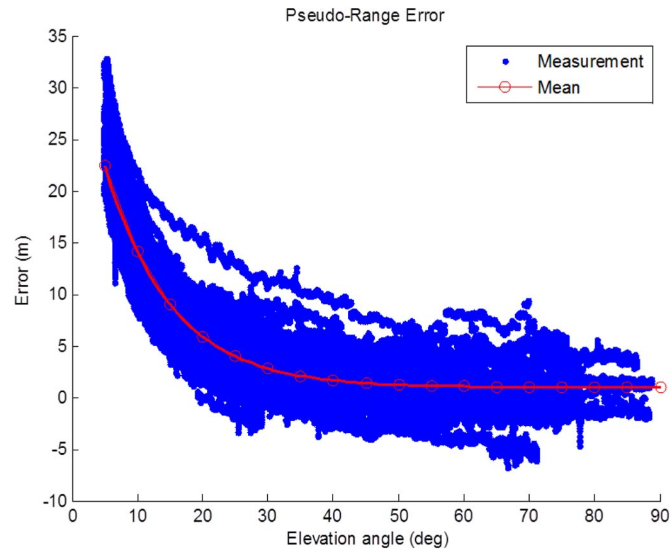

Fig. 3. Pseudorange error with respect to elevation angles.

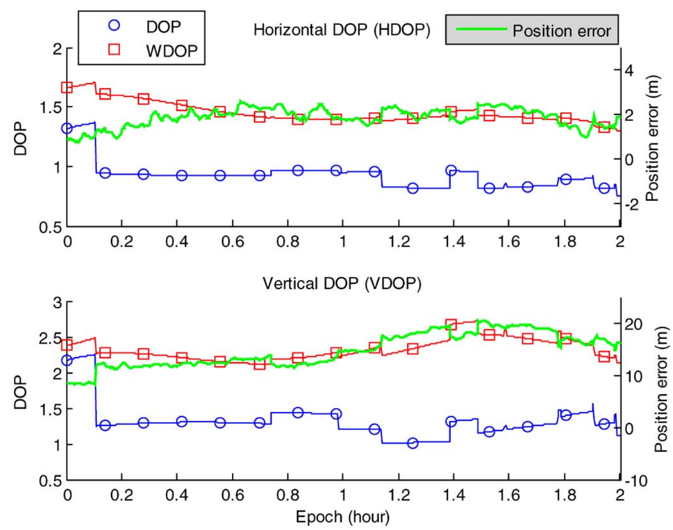

Fig. 4. Comparison of the DOP, WDOP, and position error in the horizontal and vertical directions over 2 h.

was used. The estimated error model was then inflated to reflect the trend of lower elevation angle. Equation (26) shows the estimated pseudorange error model. It has a constant term to account for the errors for high-elevation satellites and an exponential term to represent additional elevation-dependent errors. By use of the error model, measurement weight matrix W can be easily computed from (16)

$$\sigma_\theta = 5.504 + 35.26 \cdot e^{-\frac{\theta}{10.14}}. \tag{26}$$

C. Analysis Results

Fig. 4 shows the DOP, WDOP, and position errors in the horizontal and vertical directions. By displaying the data in the horizontal and vertical directions, the effects of the East–North directions are combined in the horizontal direction, and it consequently becomes difficult to identify the trend for each direction. Thus, the data were plotted for each axial direction,

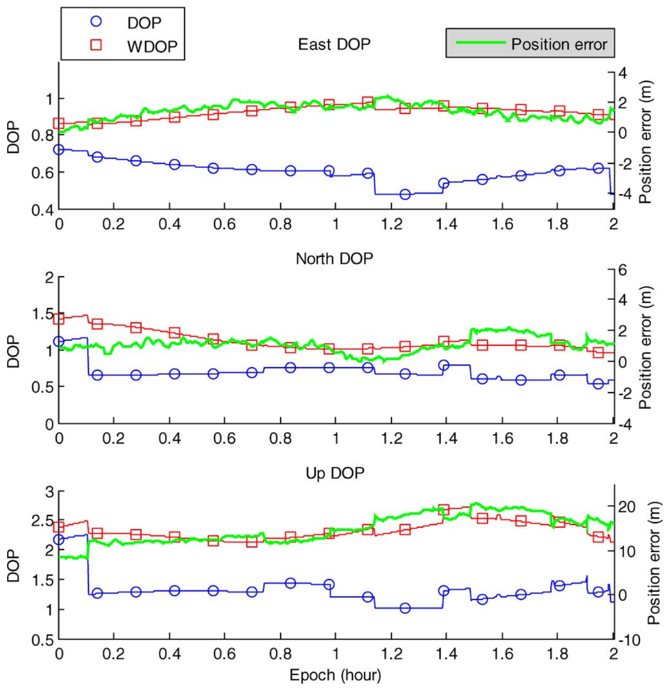

Fig. 5. Comparison of the DOP, WDOP, and position error in the East–North–Up directions over 2 h.

as shown in Fig. 5. As was shown through Theorems 1 and 2, WDOP additionally accounts for the pseudorange errors depending on the elevation angle; thus, the WDOP has a relatively larger value than the DOP and shows similar trends with the position error changes. There are intervals in the initial region of the North direction where the WDOP is not in agreement with the position error trends. However, the overall WDOP followed the position error trends. On the other hand, the conventional DOP deviated from the true position error, and there were fewer intervals where the DOP is in agreement with the position error trends. In the East DOP plot in Fig. 5, the true position error and the WDOP showed the same trend of decreasing after increasing; however, the DOP showed the opposite trend. This phenomenon was observable in the East, North, and Up directions as well. When the position error and DOP show different trends, the difference between the WDOP and DOP values was quite substantial. This difference acts as a buffer for the WDOP to reflect the effect of satellite elevation angle. In addition, the difference reflected the position error and continuously varied. As the WDOP has greater magnitude than the DOP and is in agreement with the position error trends, it can be inferred that the conventional DOP is underestimated due to its inability to reflect the effect of the pseudorange error depending on the satellite elevation angle.

In Fig. 5, the WDOP is in agreement overall with the position error around the 1.3-h region, but the DOP shows the opposite trend and greatly deviates from the WDOP. Fig. 6 is a comparison of the satellite constellation at 0.2 and 1.3 h to observe any phenomena that occur at around 1.3 h. Looking at the satellite constellation at 0.2 h, there are a total of 10 visible satellites, 1 satellite with elevation angle below 10°, and 1 satellite with an elevation angle within $10°-20°$. There are only a small number of satellites with a relatively low elevation

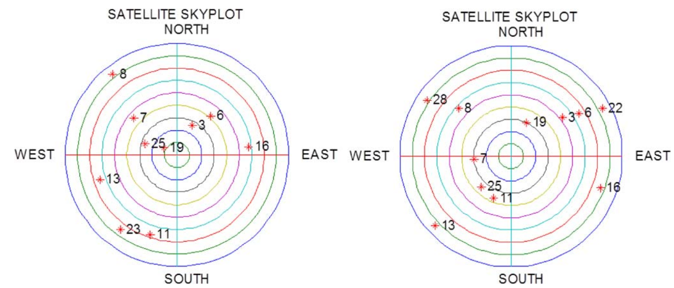

Fig. 6. Visible satellite distribution (left: 0.2 h; right: 1.3 h).

angle. Furthermore, there are no satellites in the East–South direction, and hence, the calculated DOP is predicted to have a large value. On the other hand, there are a total of 11 visible satellites, 3 satellites with elevation angle below 10°, and 1 satellite with elevation angle within $10°-20°$ at 1.3 h. At this point, there are a number of satellites with low elevation angles. Moreover, a PRN 16 satellite is in the East–South direction, and thus, the satellite constellation is improved relative to that of 0.2 h. The visible satellites at 1.3 h are widely distributed, with a satellite in all directions, and the DOP value consequently decreases. However, the position error at this time is increasing. Thus, although the geometric configuration was improved by the wide distribution of satellites, the position error increases as many low-elevation-angle satellites with large pseudorange error, as shown in Fig. 3, were included.

As the conventional DOP takes into consideration only the geometric formation, the DOP tends to decrease when comparing between the two time points. In contrast, the proposed WDOP in this paper was calculated by applying a weight that reflects the satellite elevation angle, and the proposed WDOP increases in accordance with the position error trends. This aspect reveals the limitation of the conventional DOP and the proposed WDOP's reflection of the position error trend.

Table I shows the results of the position error correlation analysis using the previously analyzed data. The correlation coefficient refers to the degree of statistical correlation that exists between two variables and has a value between -1 and 1. There are various methods to calculate the correlation coefficient. This paper used SPSS 12.0 to analyze the correlation coefficient. The Pearson, Spearman, and Kendall indices were used for the correlation coefficient. The correlation coefficients in Table I reveal that the WDOP is positively correlated with the position error, whereas the DOP has a negative correlation, meaning that it gives the opposite tendencies. Among the three axes, the East and Up directions had a position error correlation coefficient above 0.5, showing a large positive correlation. On the other hand, the position error correlation coefficient was relatively smaller in the North axis, but it changed in agreement with the position error as the Spearman and Kendall indices were positive. The tendency in the North direction was smaller than that in the other two axes because the correlation was reduced due to environmental error factors. Meanwhile, the conventional DOP has a relatively large negative correlation, showing the opposite tendency of the position error behavior. As can be seen in the overall correlation coefficient analysis, the conventional DOP has a negative correlation, whereas the

TABLE I
CORRELATION ANALYSIS RESULTS

Axial Directions	Correlation Coefficient	DOP	WDOP
East	Pearson	-0.6270	0.5888
	Spearman	-0.5013	0.5387
	Kendall	-0.5013	0.5387
North	Pearson	-0.3358	-0.0084
	Spearman	-0.4842	0.1654
	Kendall	-0.4842	0.1654
Up	Pearson	-0.6315	0.5711
	Spearman	-0.5872	0.5527
	Kendall	-0.5872	0.5527

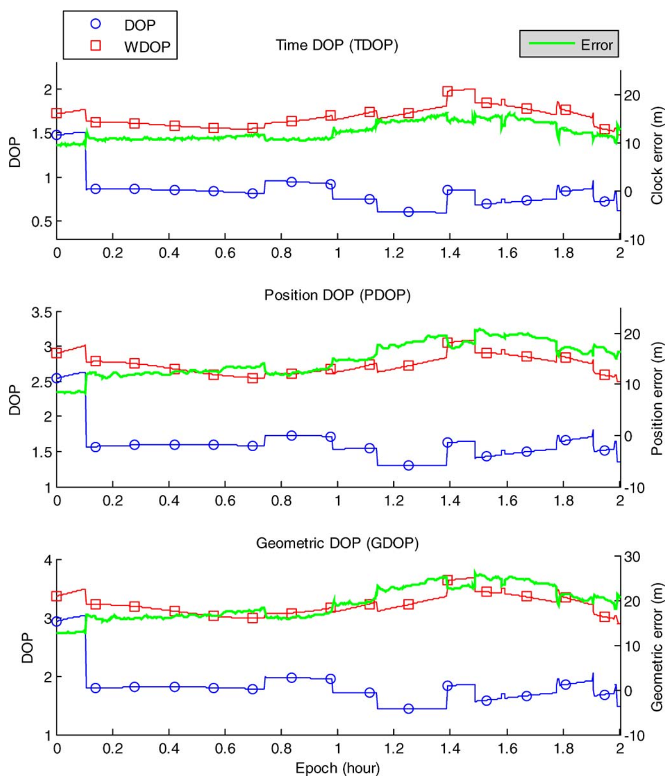

Fig. 7. TDOP, PDOP, and GDOP comparison results.

WDOP has a positive correlation, and thus, the WDOP and the position error display the same trends.

Fig. 7 presents additional comparisons of the time DOP (TDOP), the position DOP (PDOP), and the geometric DOP (GDOP), which are obtained in the process of the DOP calculations. In agreement with the axial direction analysis results, the WDOP had a similar tendency with the calculated error and had a larger magnitude than the DOP. The interval where the conventional DOP and position error have different trends corresponded, but the WDOP and position changed according to the same form.

The analysis results show that the WDOP is in greater agreement and better correlation with the position error in comparison with the DOP. In conclusion, the WDOP can be applied as a more accurate performance index as it effectively reflects the effect of satellite elevation angle.

VI. CONCLUSION

This paper has analyzed the limitation of the conventional DOP, which is a widely adopted tool for GNSS design and performance evaluation, and has proposed the WDOP that reflects the effect of satellite elevation angle change. The conventional DOP is not able to reflect the position error realistically as the pseudorange error change according to satellite elevation angle change is not considered. In this paper, the pseudorange error has been set as a function of the satellite elevation angle to calculate the weight, and WDOP has been subsequently derived in order to overcome the limitation of DOP. Mathematical analysis results showed that the WDOP had larger values than the DOP because the former additionally reflects the error that occurs as the satellite elevation angle is decreased. The conventional DOP had a lower value because the satellite elevation angle effect was not considered. Therefore, the WDOP can be utilized as a coefficient that describes the position error with greater reliability as it follows the position error trend faithfully by reflecting the satellite elevation angle change, unlike the DOP.

To verify the performance of the proposed WDOP, a comparative analysis was conducted with the conventional DOP and position error using collected data. The pseudorange error increased as the satellite elevation angle decreased, and the overall WDOP showed close agreement with the position error trends. In contrast, the conventional DOP deviated from the actual position error frequently and thus failed to show true trend of the position error. Even in the intervals where the DOP did not follow the actual position error trend, the WDOP was in better agreement with the position error. The phenomenon where the position error and DOP had opposite trends occurred when the DOP decreased due to inclusion of low-elevation-angle satellites, but the position error increased due to the increase in the pseudorange error. In this interval, the WDOP reflected the pseudorange error and thus followed the position error trend. Based on the analysis results, it is concluded that the proposed WDOP reflects the actual position error trends better than the conventional DOP.

The WDOP can be applied in allocating navigation satellites and in vision-based navigation where the subject position is analyzed through feature point allocation. Furthermore, it is expected to improve the performance prediction accuracy according to the sensor formation of a measurement system that utilizes a geometric configuration.

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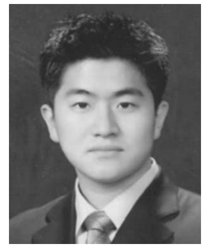

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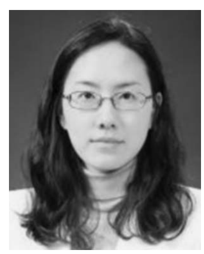

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